

JEE-MAIN EXAMINATION – JANUARY 2026

(HELD ON FRIDAY 23rd JANUARY 2026)

TIME : 3:00 PM TO 6:00 PM

MATHEMATICS

TEST PAPER WITH SOLUTIONS

SECTION-A

1. If $f(x) = \begin{cases} \frac{a|x| + x^2 - 2(\sin|x|)(\cos|x|)}{x}, & x \neq 0 \\ b, & x = 0 \end{cases}$

is continuous at $x = 0$, then $a + b$ is equal to :

- (1) 1 (2) 2
(3) 0 (4) 4

Ans. (2)

Sol. $f(x) = \begin{cases} \frac{a|x| + x^2 - 2\sin|x|\cos|x|}{x}; & x \neq 0 \\ b; & x = 0 \end{cases}$

for continuity

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = f(0)$$

$$\lim_{x \rightarrow 0^-} \frac{ah + h^2 - 2(\sinh)\cosh}{-h}$$

$$= \lim_{x \rightarrow 0^+} \frac{ah + h^2 - 2(\sinh)\cosh}{h}$$

$$\text{or } -a + 2 = a - 2 = b$$

$$2a = 4$$

$$a = 2, b = 0$$

$$\therefore a + b = 2$$

2. Let $\frac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\frac{1}{2\sqrt{2}}$. Then the value of

$$\sin\left(\frac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta) + \cos\left(\frac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$$

is equal to :

- (1) $\frac{1-\sqrt{2}}{\sqrt{3}}$ (2) $-\frac{\sqrt{2}}{\sqrt{3}}$
(3) $\frac{\sqrt{2}-1}{\sqrt{3}}$ (4) $\frac{\sqrt{2}}{\sqrt{3}}$

Ans. (1)

Sol. $\frac{\pi}{2} < \theta < \pi$ and $\cot\theta = -\frac{1}{2\sqrt{2}}$

$$\Rightarrow \sin\left(\frac{15\theta}{2}\right)(\cos 8\theta + \sin 8\theta)$$

$$+ \cos\left(\frac{15\theta}{2}\right)(\cos 8\theta - \sin 8\theta)$$

$$\Rightarrow \sin\left(\frac{15\theta}{2}\right)\cos 8\theta - \cos\left(\frac{15\theta}{2}\right)\sin 8\theta$$

$$+ \sin\frac{15\theta}{2}\sin 8\theta + \cos\frac{15\theta}{2}\cos 8\theta$$

$$\Rightarrow \sin\left(\frac{15\theta}{2} - 8\theta\right) + \cos\left(\frac{15\theta}{2} - 8\theta\right)$$

$$\Rightarrow \cos\frac{\theta}{2} - \sin\frac{\theta}{2} = -\sqrt{1 - \sin\theta} \quad \left(\frac{\pi}{4} < \frac{\theta}{2} < \frac{\pi}{2}\right)$$

$$\Rightarrow \text{given } \cot\theta = -\frac{1}{2\sqrt{2}}, \sin\theta = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow -\sqrt{1 - \sin\theta} = \sqrt{\frac{3 - 2\sqrt{2}}{3}} = -\frac{(\sqrt{2} - 1)}{3}$$

$$= \frac{1 - \sqrt{2}}{\sqrt{3}}$$

3. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$ and $\vec{v} = \vec{a} \times \vec{b}$. If $\vec{v} \cdot \vec{c} = 11$ and the length of the projection of $\vec{b}$ on $\vec{c}$ is p , then $9p^2$ is equal to :

- (1) 9 (2) 6
(3) 4 (4) 12

Ans. (4)



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Sol. $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{c} = \lambda\hat{i} + \hat{j} + \hat{k}$, and

$$\vec{v} = \vec{a} \times \vec{b}. \text{ If } \vec{v} \cdot \vec{c} = 11$$

$$\vec{v} = (\vec{a} \times \vec{b}) = (-\hat{i} + 7\hat{j} + 5\hat{k})$$

$$\vec{v} \cdot \vec{c} = 11 = (-\hat{i} + 7\hat{j} + 5\hat{k}) \cdot (\lambda\hat{i} + \hat{j} + \hat{k}) = 11$$

$$\Rightarrow -\lambda + 7 + 5 = 11$$

$$\Rightarrow \lambda = 1$$

Length of projection of $\vec{b}$ on $\vec{c} = \vec{b} \cdot \hat{c}$

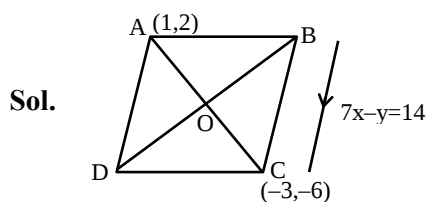
$$\Rightarrow \left| (2\hat{i} + \hat{j} - \hat{k}) \cdot \frac{(\hat{i} + \hat{j} + \hat{k})}{\sqrt{3}} \right| = \frac{2+1-1}{\sqrt{3}} = P = \frac{2}{\sqrt{3}}$$

$$\Rightarrow 9p^2 = 9\left(\frac{4}{3}\right) = 12$$

4. Let A(1, 2) and C(-3, -6) be two diagonally opposite vertices of a rhombus, whose sides AD and BC are parallel to the line $7x - y = 14$. If B(α , β) and D(γ , δ) are the other two vertices, then $|\alpha + \beta + \gamma + \delta|$ is equal to :

- (1) 9 (2) 3
(3) 6 (4) 1

Ans. (3)



Given the points of B and D are (α , β) and (γ , δ) and mid point of A and C is (-1, -2)

$$\text{So } \frac{\alpha + \gamma}{2} = -1 \text{ and } \frac{\beta + \delta}{2} = -2$$

$$|\alpha + \gamma + \beta + \delta| = 6$$

5. Consider two sets $A = \{x \in \mathbb{Z} : (|x - 3| - 3) \leq 1\}$ and $B = \left\{x \in \mathbb{R} - \{1, 2\} : \frac{(x-2)(x-4)}{x-1} \log_e(|x-2|) = 0\right\}$.

Then the number of onto functions $f : A \rightarrow B$ is equal to :

- (1) 62 (2) 79
(3) 32 (4) 81

Ans. (1)

Sol. $A : |x - 3| - 3 \leq 1$

$$\Rightarrow -1 \leq |x - 3| - 3 \leq 1$$

$$2 \leq |x - 3| \leq 4$$

$$2 \leq (x - 3) \leq 4 \text{ or } -4 \leq (x - 3) \leq -2$$

$$5 \leq x \leq 7 \text{ or } -1 \leq x \leq 1$$

$$A = \{-1, 0, 1, 5, 6, 7\}$$

$$B \Rightarrow x = 4, |x - 2| = 1 \Rightarrow x = 3 \text{ or } 1 (\text{reject}) +$$

$$B = \{3, 4\}$$

$$\text{Number of onto functions from A to B} = 2^6 - 2 = 62$$

6. The system of linear equations

$$x + y + z = 6$$

$$2x + 5y + az = 36$$

$$x + 2y + 3z = b$$

has

- (1) unique solution for $a = 8$ and $b = 16$
(2) infinitely many solutions for $a = 8$ and $b = 14$
(3) infinitely many solutions for $a = 8$ and $b = 16$
(4) unique solution for $a = 8$ and $b = 14$

Ans. (2)

Sol. If $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & a \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow a = 8$

$$\text{If } D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 36 & 5 & a \\ b & 2 & 3 \end{vmatrix} = 0 \Rightarrow ab - 5b - 12a + 54 = 0$$

$$\text{If } D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 2 & 36 & a \\ 1 & b & 3 \end{vmatrix} = 0 \Rightarrow ab - 6a - 2b - 36 = 0$$

$$\text{If } D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 5 & 36 \\ 1 & 2 & b \end{vmatrix} = 0 \Rightarrow b = 14$$

For $a = 8$ & $b = 14 \Rightarrow D_1$ & D_2 are also zero

For $a = 8$ & $b = 14 \Rightarrow D = D_1 = D_2 = D_3 = 0$
 $\Rightarrow$ infinitely many solutions.



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7. The sum of all the real solutions of the equation $\log_{(x+3)}(6x^2 + 28x + 30) = 5 - 2\log_{(6x+10)}(x^2 + 6x + 9)$ is equal to :

- (1) 2 (2) 1
(3) 0 (4) 4

Ans. (3)

Sol. $\log_{x+3}[(x+3)(6x+10)] = 5 - 2\log_{6x+10}(x+3)^2$

$$1 + \log_{x+3}(6x+10) = 5 - 4\log_{6x+10}(x+3)$$

$$\text{Let } \log_{(x+3)}(6x+10) = A$$

$$\Rightarrow A + \frac{4}{A} = 4 \text{ or } A = 2$$

$$\Rightarrow \log_{(x+3)}(6x+10) = 2$$

$$\Rightarrow 6x+10 = (x+3)^2$$

$$\Rightarrow 6x+10 = x^2 + 9 + 6x$$

$$\Rightarrow x^2 = 1, x = \pm 1$$

So sum of roots = 0

8. If the points of intersection of the ellipses $x^2 + 2y^2 - 6x - 12y + 23 = 0$ and $4x + 2y^2 - 20x - 12y + 35 = 0$ lie on a circle of radius r and centre (a, b) , then the value of $ab + 18r^2$ is

- (1) 53 (2) 51
(3) 52 (4) 55

Ans. (4)

Sol. By family of curve equation of circle will be

$$\Rightarrow S_1 + \lambda S_2 = 0$$

$$\Rightarrow (x^2 + 2y^2 - 6x - 12y + 23)$$

$$+ \lambda(4x^2 + 2y^2 - 20x - 12y + 35) = 0$$

$$\Rightarrow \text{for circle coeff of } x^2 = \text{coeff. of } y^2$$

$$\Rightarrow \lambda = \frac{1}{2}$$

So equation of circle is

$$\Rightarrow x^2 + y^2 - \frac{16}{3}x - 6y + \frac{27}{2} = 0$$

$$\text{Centre } \left(\frac{8}{3}, 3\right) : \text{Radius } r = \sqrt{\frac{47}{18}} = r$$

$$\therefore ab + 18r^2 = 8 + 47 = 55$$

9. Let PQ be a chord of the hyperbola $\frac{x^2}{4} - \frac{y^2}{b^2} = 1$,

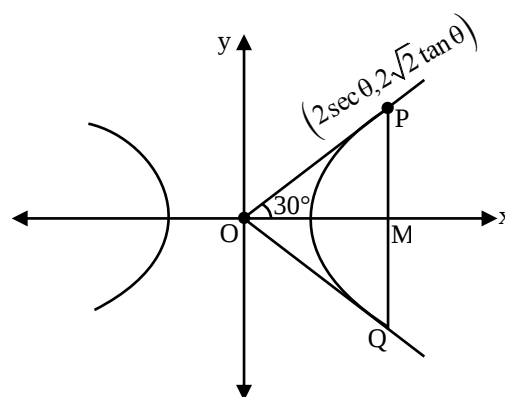
perpendicular to the x-axis such that OPQ is an equilateral triangle, O being the centre of the hyperbola. If the eccentricity of the hyperbola is $\sqrt{3}$, then the area of the triangle OPQ is :

- (1) $2\sqrt{3}$ (2) $\frac{8\sqrt{3}}{5}$
(3) $\frac{11}{5}$ (4) $\frac{9}{5}$

Ans. (2)

$$\text{Sol. } e = \sqrt{1 + \frac{b^2}{4}} = \sqrt{3} \Rightarrow b = 8$$

$$\therefore \text{Hyperbola } \frac{x^2}{4} - \frac{y^2}{8} = 1$$



$$\frac{PM}{OM} = \tan 30^\circ$$

$$\Rightarrow \frac{2\sqrt{2} \tan \theta}{2 \sec \theta} = \frac{1}{\sqrt{3}} \Rightarrow \sin \theta = \frac{1}{\sqrt{6}}$$

$$\text{Area} = 2 \times \frac{1}{2} \times OM \times MP$$

$$= 2 \sec \theta \times 2\sqrt{2} \tan \theta$$

$$= 4\sqrt{2} \frac{\sin \theta}{\cos^2 \theta} = 4\sqrt{2} \times \frac{1}{\sqrt{6} \times \left(1 - \frac{1}{6}\right)} = \frac{8\sqrt{3}}{5}$$



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10. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $\vec{a} \times \vec{b} = 2(\vec{a} \times \vec{c})$. If $|\vec{a}| = 1, |\vec{b}| = 4, |\vec{c}| = 2$, and the angle between $\vec{b}$ and $\vec{c}$ is 60° , then $|\vec{a} \cdot \vec{c}|$ is :
- (1) 2 (2) 4
(3) 0 (4) 1

Ans. (4)

Sol. $\vec{a} \times \vec{b} - 2(\vec{a} \times \vec{c}) = 0$

$$\vec{a} \times (\vec{b} - 2\vec{c}) = 0 \Rightarrow \vec{b} - 2\vec{c} = \lambda \vec{a} \quad \dots(1)$$

$$|\lambda \vec{a}|^2 = |\vec{b} - 2\vec{c}|^2 \Rightarrow \lambda^2 |\vec{a}|^2 = b^2 + 4c^2 - 4\vec{b} \cdot \vec{c}$$

$$\lambda^2 = 16 + 16 - 4 \cdot 4 \cdot 2 \cdot \frac{1}{2}$$

$$\lambda^2 = 16$$

$$\lambda = \pm 4$$

$$\therefore \vec{b} - 2\vec{c} = \pm 4\vec{a}$$

$$\text{Dot with } \vec{c} \Rightarrow \vec{b} \cdot \vec{c} - 2|\vec{c}|^2 = \pm 4(\vec{a} \cdot \vec{c})$$

$$4 \cdot 2 \cdot \frac{1}{2} - 2 \cdot 4 = \pm 4(\vec{a} \cdot \vec{c})$$

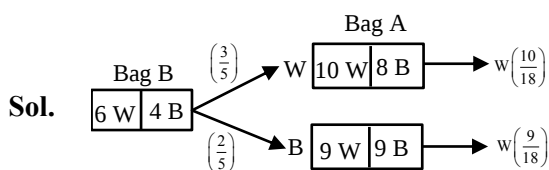
$$|\vec{a} \cdot \vec{c}| = 1$$

11. Bag A contains 9 white and 8 black balls, while bag B contains 6 white and 4 black balls. One ball is randomly picked up from the bag B and mixed up with the balls in the bag A. Then a ball is randomly drawn from the bag A. If the probability, that the ball drawn is white, is $\frac{p}{q}$, $\gcd(p, q) = 1$,

then $p + q$ is equal to

- (1) 22 (2) 23
(3) 24 (4) 21

Ans. (2)



$$\therefore P(\text{Drawn ball is white}) = \frac{3}{5} \times \frac{10}{18} + \frac{2}{5} \times \frac{9}{18}$$

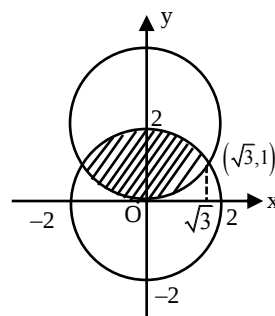
$$= \frac{48}{90} = \frac{8}{15} = \frac{p}{q}$$

$$\therefore p + q = 23$$

12. The area of the region enclosed between the circles $x^2 + y^2 = 4$ and $x^2 + (y - 2)^2 = 4$ is :

- (1) $\frac{2}{3}(2\pi - 3\sqrt{3})$ (2) $\frac{4}{3}(2\pi - 3\sqrt{3})$
(3) $\frac{4}{3}(2\pi - \sqrt{3})$ (4) $\frac{2}{3}(4\pi - 3\sqrt{3})$

Ans. (4)



Sol.

$$A = 2 \int_0^{\sqrt{3}} \left[\sqrt{4-x^2} - (2 - \sqrt{4-x^2}) \right] dx$$

$$= 2 \int_0^{\sqrt{3}} (2\sqrt{4-x^2} - 2) dx$$

$$= 4 \int_0^{\sqrt{3}} (\sqrt{4-x^2} - 1) dx$$

$$= \left[4 \left[\frac{1}{2} \left(x\sqrt{4-x^2} + 4\sin^{-1} \frac{x}{2} \right) - x \right] \right]_0^{\sqrt{3}}$$

$$= 4 \left[\frac{1}{2} \left(\sqrt{3} + 4 \times \frac{\pi}{3} \right) - \sqrt{3} \right] = 4 \left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right]$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \text{ (Sq. units)}$$

13. The least value of $(\cos^2 \theta - 6\sin \theta \cos \theta + 3\sin^2 \theta + 2)$ is

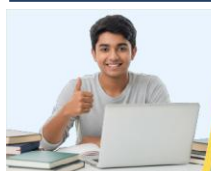
- (1) -1 (2) $4 + \sqrt{10}$
(3) $4 - \sqrt{10}$ (4) 1

Ans. (3)

Sol. $f(\theta) = \frac{1 + \cos 2\theta}{2} - 3\sin 2\theta + 3 \left(\frac{1 - \cos 2\theta}{2} \right) + 2$

$$f(\theta) = 4 - 3\sin 2\theta - \cos 2\theta$$

$$f(\theta) \in [4 - \sqrt{10}, 4 + \sqrt{10}]$$



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14. Let $I(x) = \int \frac{3dx}{(4x+6)(\sqrt{4x^2+8x+3})}$ and

$I(0) = \frac{\sqrt{3}}{4} + 20$. If $I\left(\frac{1}{2}\right) = \frac{a\sqrt{2}}{b} + c$, where a, b, c

$\in \mathbb{N}$, $\gcd(a, b) = 1$, then $a + b + c$ is equal to :

- (1) 29 (2) 28
(3) 31 (4) 30

Ans. (3)

Sol. Let $4x + 6 = \frac{1}{t} \Rightarrow x = \frac{1-t}{4}$

$$4dx = -\frac{dt}{t^2} \quad \left\{ \begin{array}{l} x+1 = \frac{1-t}{4} - 2 \\ \frac{1-t}{4} \end{array} \right.$$

$$\int \frac{3dx}{(4x+6)\sqrt{4(x+1)^2-1}}$$

$$\int \frac{3(-dt)}{4t^2 \times \frac{1}{t} \sqrt{4\left(\frac{1-t}{4}\right)^2 - 1}}$$

$$-\frac{3}{4} \int \frac{dt}{t \sqrt{\frac{(1-2t)^2}{4t^2} - 1}}$$

$$-\frac{3}{4} \int \frac{dt(2t)}{t \sqrt{1-4t}}$$

$$-\frac{3}{2} \int \frac{dt}{\sqrt{1-4t}} = -\frac{3}{2} \left(\frac{\sqrt{1-4t}}{\frac{1}{2} \times -4} \right) + c$$

$$= \frac{3}{4} \sqrt{1-4t} + c \quad \because t = \frac{1}{4x+6}$$

$$= \frac{3}{4} \sqrt{1-4\left(\frac{1}{4x+6}\right)} + c$$

$$= \frac{3}{4} \sqrt{\frac{4x+6-4}{4x+6}} + c$$

$$I(x) = \frac{3}{4} \sqrt{\frac{4x+2}{4x+6}} + c$$

$$I(0) = \frac{3}{4} \sqrt{\frac{2}{6}} + c$$

$$I(0) = \frac{\sqrt{3}}{4} + c \Rightarrow c = 20$$

$$\text{Hence } I(x) = \frac{3}{4} \sqrt{\frac{4x+2}{4x+6}} + 20$$

$$I\left(\frac{1}{2}\right) = \frac{3}{4} \sqrt{\frac{4}{8}} + 20$$

$$= \frac{3}{4\sqrt{2}} + 20$$

$$= \frac{3\sqrt{2}}{8} + 20$$

$$a + b + c = 3 + 8 + 20 = 31$$

15. The number of ways, in which 16 oranges can be distributed to four children such that each child gets at least one orange, is

- (1) 429 (2) 384
(3) 403 (4) 455

Ans. (4)

Sol. Let oranges are identical then

$$x_1 + x_2 + x_3 + x_4 = 16$$

$$\text{and } x_1, x_2, x_3, x_4 \geq 1$$

$$\text{or } x'_1 + x'_2 + x'_3 + x'_4 = 12$$

so total number of solutions are

$$= {}^{12+3}C_3 = {}^{15}C_3 = 455$$

16. Let $A = \{0, 1, 2, \dots, 9\}$. Let R be a relation on A defined by $(x, y) \in R$ if and only if $|x-y|$ is a multiple of 3.

Given below are two statements:

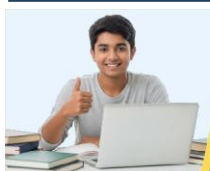
Statement I: $n(R) = 36$

Statement II: R is an equivalence relation.

In the light of the above statements, choose the correct answer from the options given below

- (1) Both Statement I and Statement II are correct
(2) Statement I is incorrect but Statement II is correct
(3) Statement I is correct but Statement II is incorrect
(4) Both Statement I and Statement II are incorrect

Ans. (2)



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Sol. Number of form $3K = 4$
 Number of form $3K + 1 = 3$
 Number of form $3K + 2 = 3$
 $4 \times 4 + 3 \times 3 + 3 \times 3 = 34$ relations
 $\Rightarrow x R y \Rightarrow y R x$
 $\Rightarrow (x - y) = 3\lambda, (y - z) = 3\mu$
 $\Rightarrow (x - z) = 3(\lambda + \mu)$
 R is reflexive, symmetric and transitive S_2 is true
 Ans. S_1 is false but S_2 is true

17. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}$, $i = \sqrt{-1}$, then $(z^{201} - i)^8$ is equal to
 (1) -1 (2) 0
 (3) 1 (4) 256

Ans. (4)

Sol. $z = \cos \frac{\pi}{6} + i \sin \frac{\pi}{6}$
 $z^{201} = \cos \left(201 \frac{\pi}{6} \right) + i \sin \left(201 \frac{\pi}{6} \right) = -i$
 $(z^{201} - i)^8 = (-2i)^8 = 256$

18. If the mean and the variance of the data

Class	4-8	8-12	12-16	16-20
Frequency	3	λ	4	7

are μ and 19 respectively, then the value of $\lambda + \mu$ is

- (1) 18 (2) 21
 (3) 20 (4) 19

Ans. (4)

Sol. $\mu = \frac{\sum f_i x_i}{\sum f_i} = \frac{18 + 10\lambda + 56 + 126}{14 + \lambda}$

$$= \frac{200 + 10\lambda}{\lambda + 14} = 10 + \left(\frac{60}{\lambda + 14} \right)$$

$\lambda + 14$ is multiply of 60 $\Rightarrow \lambda = 1$ or 6 or 16.

$$\sigma^2 = \frac{\sum x_i^2}{\lambda + 14} - (\mu)^2$$

$$\sigma^2 = \frac{6^2(3) + 10^2(\lambda) + 14^2(4) + (18)^2(7)}{\lambda + 14} - \mu^2$$

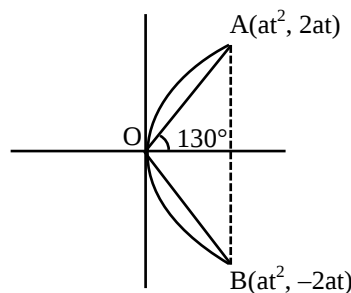
for $\lambda = 6$, $\mu = 10 + 3 = 13$

$\lambda + \mu = 19$

19. An equilateral triangle OAB is inscribed in the parabola $y^2 = 4x$ with the vertex O at the vertex of the parabola. Then the minimum distance of the circle having AB as a diameter from the origin is

- (1) $4(3 - \sqrt{3})$ (2) $2(8 - 3\sqrt{3})$
 (3) $4(6 + \sqrt{3})$ (4) $2(3 + \sqrt{3})$

Ans. (1)



Sol.

$$M_{OA} = \frac{2t - 0}{t^2 - 0} = \frac{2}{t}$$

$$\frac{2}{t} = \tan 30^\circ$$

$$t = 2\sqrt{3}$$

$$\text{Req. Circle : } (x - 12)^2 + y^2 = (4\sqrt{3})^2$$

$$\text{Least distance} = |CP - R|$$

$$= |2 - 4\sqrt{3}| = 4(3 - \sqrt{3})$$

20. Let $\sum_{k=1}^n a_k = \alpha n^2 + \beta n$. If $a_{10} = 59$ and $a_6 = 7a_1$, then

$\alpha + \beta$ is equal to

- (1) 12 (2) 3
 (3) 5 (4) 7

Ans. (3)

Sol. $a_n = S_n - S_{n-1}$
 $= (\alpha n^2 + \beta n) - (\alpha(n-1)^2 + \beta(n-1))$

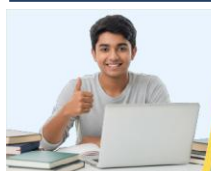
$$(1) a_{10} \Rightarrow 19\alpha + \beta = 59$$

$$(2) a_6 = 7a_1 \Rightarrow 11\alpha + \beta = 7(\alpha + \beta)$$

$$\Rightarrow 2\alpha = 3\beta$$

$$a = 3, \beta = 2$$

$$\boxed{\alpha + \beta = 5}$$



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SECTION-B

21. If the solution curve $y = f(x)$ of the differential equation $(x^2 - 4)y' - 2xy + 2x(4 - x^2)^2 = 0$ $x > 2$, passes through the point $(3, 15)$, then the local maximum value of f is

Ans. (16)

Sol. $(x^2 - 4)y' - 2xy + 2x(4 - x^2)^2 = 0$

$$d\left(\frac{y}{x^2 - 4}\right) = -2x$$

$$y = (-x^2 + C)(x^2 - 4)$$

$$\text{for } x = 3, y = 15 \Rightarrow C = 12$$

$$y = (-x^2 + 12)(x^2 - 4)$$

$$y' = 0 \Rightarrow x = 2\sqrt{2}$$

$$y_{\text{local max}} = \left((2\sqrt{2})^2 - 4\right)\left(-(2\sqrt{2})^2 + 12\right)$$

$$= 16$$

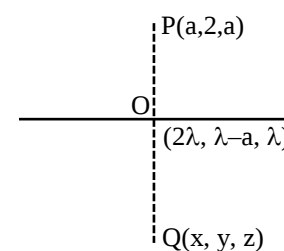
22. If the image of the point $P(a, 2, a)$ in the line

$$\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1} \text{ is } Q \text{ and the image of } Q \text{ in the}$$

$$\text{line } \frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5} \text{ is } P, \text{ then } a + b \text{ is}$$

equal to.....

Ans. (3)

Sol. 

$$\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$$

$$Q. (4\lambda - a, 2\lambda - 2a - 2, 2\lambda - a)$$

Now,

$$\frac{x-2b}{2} = \frac{y-a}{1} = \frac{z+2b}{-5}$$

$(2\mu + 2b, \mu + a, -5\mu - 2b)$

$P(a, 2, a)$

$$\frac{2\lambda - a + a}{2} = -5\mu - 2b$$

$$\text{and } \frac{a + 4\lambda - a}{2} = 2\mu + 2b$$

$$\text{and } \frac{2\lambda - 2a - 2 + 2}{2} = \mu + a$$

$$\Rightarrow \lambda - \mu = b$$

$$\lambda - \mu = 2a$$

$$\lambda + 5\mu = -2b$$

$$\Rightarrow b = 2a \text{ and } \lambda = a, \mu = -a$$

$P(a, 2, a)$

$$\frac{x}{2} = \frac{y+a}{1} = \frac{z}{1}$$

$2\hat{i} + \hat{j} + \hat{k}$

$(2\mu, \lambda - a, \lambda)$

For $\lambda = a$

$(2a, 0, a)$

$$(a\hat{i} - 2\hat{j} + 0\hat{k}) \cdot (2\hat{i} + \hat{j} + \hat{k}) = 0$$

$$2a - 2 = 0$$

$$a = 1$$

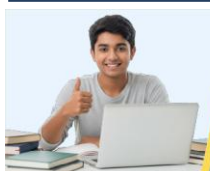
$$b = 2a = 2 \times 1 = 2$$

$$a + b = 1 + 2 = 3$$

23. The number of elements in the

$$\text{set } S = \left\{ x : x \in [0, 100] \text{ and } \int_0^x t^2 \sin(x-t) dt = x^2 \right\} \text{ is } \dots$$

Ans. (16)



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Sol. $\int_0^x t^2 \sin(x-t) dt = x^2$

Use by parts

$$-x^2 \cos x + x^2 + 2x^2 \cos x - 2x \sin x$$

$$-x^2 \cos x + 2x \sin x + 2 \cos x - 2 = x$$

$$\cos x = 1$$

$$x = 0, 2\pi, 4\pi, \dots, 30\pi$$

$$\text{Total Elements} = 16$$

24. Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -2 & 0 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and B be a matrix such that

$B(I - A) = I + A$. Then the sum of the diagonal elements of $B^T B$ is equal to _____.

Ans. (3)

Sol. $A^T = -A$

$$B = (I + A)(I - A)^{-1}$$

$$B^T = ((I - A)^{-1})^T (I + A)^T$$

$$B^T = (I - A^T)^{-1} (I + A^T)$$

$$B^T = (I + A)^{-1} (I + A)$$

$$B^T B = (I + A)^{-1} (I - A)(I + A)(I - A)^{-1}$$

$$= (I + A)^{-1} (I + A)(I - A)(I - A)^{-1}$$

$$= I$$

$$\text{tr}(B^T B) = 3$$

25. Let S denote the set of 4-digit numbers abcd such that $a > b > c > d$ and P denote the set of 5-digit numbers having product of its digits equal to 20. Then $n(S) + n(P)$ is equal to.....

Ans. (260)

Sol. For $n(s) = {}^{10}C_4 = 210$

$$(5, 4, 1, 1, 1) \quad (5, 2, 2, 1, 1)$$

$$\text{For } n(p) = \frac{5!}{3!} + \frac{5!}{2!2!} = 50$$

$$n(s) + n(p) = 210 + 50 = 260$$



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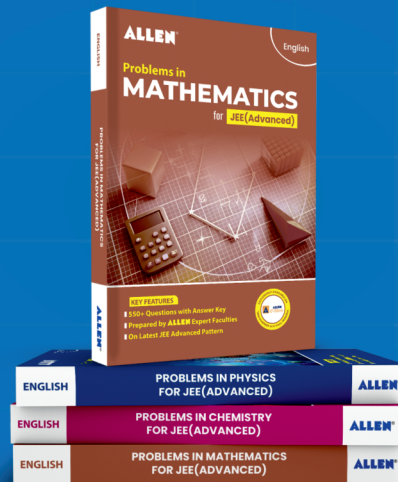
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